

## GATE 2014: General Instructions during Examination

1. Total duration of the GATE examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Common Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use a non-programmable type calculator, however, sharing of calculators is not allowed.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

 You have not visited the question yet.

 You have not answered the question.

 You have answered the question.

 You have NOT answered the question, but have marked the question for review.

 You have answered the question, but marked it for review.

The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is answered, but marked for review, then the answer will be considered for evaluation unless the status is modified by the candidate.*

### Navigating to a Question :

7. To answer a question, do the following:
  - a. Click on the question number in the Question Palette to go to that question directly.
  - b. Select an answer for a multiple choice type question by clicking on the bubble placed before the 4 choices, namely A, B, C and D. Use the virtual numeric keypad to enter a number as answer for a numerical type question.
  - c. Click on **Save & Next** to save your answer for the current question and then go to the next question.
  - d. Click on **Mark for Review & Next** to save your answer for the current question and also mark it for review, and then go to the next question.

**Caution:** Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

### Answering a Question :

8. Procedure for answering a multiple choice (MCQ) type question:
  - a. Choose one answer from the 4 options (A,B,C,D) given below the question, click on the bubble placed before the chosen option.
  - b. To deselect your chosen answer, click on the bubble of the chosen option again or click on the **Clear Response** button.
  - c. To change your chosen answer, click on the bubble of another option.
  - d. To save your answer, you **MUST** click on the **Save & Next** button.
9. Procedure for answering a numerical answer type question:
  - a. To enter a number as your answer, use the virtual numerical keypad.
  - b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.
  - c. To clear your answer, click on the **Clear Response** button.
  - d. To save your answer, you **MUST** click on the **Save & Next** button
10. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ) or entered (for numerical answer type) for a question that is **Marked for Review**, that answer will be considered in the evaluation unless the status is modified by the candidate.*
11. To change your answer to a question that has already been answered, first select that question for answering and then follow the procedure for answering that type of question.
12. Note that **ONLY** Questions for which answers are *saved* or *marked for review after answering* will be considered for evaluation.

### Choosing a Section :

13. Sections in this question paper are displayed on the top bar of the screen. Questions in a Section can be viewed by clicking on the name of that Section. The Section you are currently viewing will be highlighted.
14. A checkbox is displayed for every optional Section, if any, in the Question Paper. To select the optional Section for answering, click on the checkbox for that Section.
15. If the checkbox for an optional Section is not selected, the **Save & Next** button and the **Mark for Review & Next** button will **NOT** be enabled for that Section. You will

only be able to see questions in this Section, but you will not be able to answer questions in the Section.

16. After clicking the **Save & Next** button for the last question in a Section, you will automatically be taken to the first question of the next Section in sequence.
17. You can move the mouse cursor over the name of a Section to view the answering status for that Section.

### **Changing the Optional Section :**

18. After answering the chosen optional Section, partially or completely, you can change the optional Section by selecting the checkbox for a new Section that you want to attempt. A warning message will appear along with a table showing the number of questions answered in each of the previously chosen optional Sections and a checkbox against each of these Sections. Click on a checkbox against a Section that you want to reset and then click on the **RESET** button. Note that RESETTING a Section will DELETE all the answers for questions in that Section. Hence, if you think that you may want to select this Section again later, you will have to note down your answers for questions in that Section. If you do not want to reset the Section and want to continue answering the previously chosen optional Section, then click on the **BACK** button.
19. If you deselect the checkbox for an optional Section in the top bar, the following warning message will appear: "Deselecting the checkbox will DELETE all the answers for questions in this Section. Do you want to deselect this Section?" If you want to deselect, click on the **RESET** button. If you do not want to deselect, click on the **BACK** button.
20. You can shuffle between different Sections or change the optional Sections any number of times.

# GATE 2014 Examination

## PH: Physics

Duration: 180 minutes

Maximum Marks: 100

### Read the following instructions carefully.

1. To login, enter your Registration Number and password provided to you. Kindly go through the various symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **View All Questions** button in the screen.
3. This question paper consists of **2 sections**, General Aptitude (GA) for **15 marks** and the subject specific GATE paper for **85 marks**. Both these sections are compulsory.  
The GA section consists of **10** questions. Question numbers 1 to 5 are of 1-mark each, while question numbers 6 to 10 are of 2-mark each.  
The subject specific GATE paper section consists of **55** questions, out of which question numbers 1 to 25 are of 1-mark each, while question numbers 26 to 55 are of 2-mark each.
4. Depending upon the GATE paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Common Data** button that appears at the top, right hand side of the screen.
5. The computer allotted to you at the examination center runs specialized software that permits only one answer to be selected for multiple-choice questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual keyboard and mouse.
6. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
7. In each paper a candidate can answer a total of 65 questions carrying 100 marks.
8. The question paper may consist of questions of **multiple choice type (MCQ)** and **numerical answer type**.
9. Multiple choice type questions will have four choices against A, B, C, D, out of which only **ONE** is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
10. For numerical answer type questions, each question will have a numerical answer and there will not be any choices. **For these questions, the answer should be entered** by using the virtual keyboard that appears on the monitor and the mouse.
11. All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of  $\frac{1}{3}$  marks for a 1-mark question and  $\frac{2}{3}$  marks for a 2-mark question.
12. There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.
13. Non-programmable type Calculator is allowed. Charts, graph sheets, and mathematical tables are **NOT** allowed in the Examination Hall. You must use the Scribble pad provided to you at the examination centre for all your rough work. The Scribble Pad has to be returned at the end of the examination.

### Declaration by the candidate:

“I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition”.

**Q. 1 – Q. 5 carry one mark each.**

- Q.1 A student is required to demonstrate a high level of comprehension of the subject, especially in the social sciences.

The word closest in meaning to comprehension is

- (A) understanding (B) meaning (C) concentration (D) stability

- Q.2 Choose the most appropriate word from the options given below to complete the following sentence.

One of his biggest \_\_\_\_\_ was his ability to forgive.

- (A) vice (B) virtues (C) choices (D) strength

- Q.3 Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

- (A) Rajan has decided to work only in a group.  
 (B) Rajan and Sajan were formed into a group against their wishes.  
 (C) Sajan had decided to give in to Rajan's request to work with him.  
 (D) Rajan had believed that Sajan and he would be working together.

- Q.4 If  $y = 5x^2 + 3$ , then the tangent at  $x = 0, y = 3$

- (A) passes through  $x = 0, y = 0$  (B) has a slope of +1  
 (C) is parallel to the  $x$ -axis (D) has a slope of -1

- Q.5 A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs 800Q, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?

**Q. 6 – Q. 10 carry two marks each.**

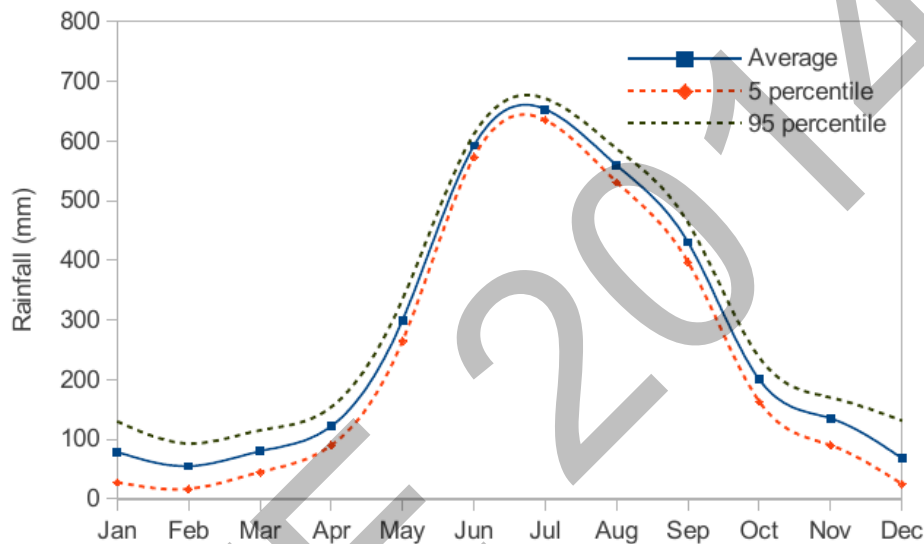
- Q.6 Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK

- (A) ALRVX (B) EPVZB (C) ITZDF (D) OYEIK

- Q.7 Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

- Q.8 The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3:4:5:6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?
- Q.9 One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?
- (A) 3.0 (B) 2.5 (C) 1.5 (D) 1.25
- Q.10 The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? ( $k$  percentile is the value such that  $k$  percent of the data fall below that value)



- (i) On average, it rains more in July than in December  
(ii) Every year, the amount of rainfall in August is more than that in January  
(iii) July rainfall can be estimated with better confidence than February rainfall  
(iv) In August, there is at least 500 mm of rainfall
- (A) (i) and (ii) (B) (i) and (iii)  
(C) (ii) and (iii) (D) (iii) and (iv)

**END OF THE QUESTION PAPER**

**Some Useful Constants**

Speed of light in free space

Boltzmann constant

Planck's constant

Electron charge

Permittivity of free space

Permeability of free space

Mass of electron

Mass of proton

Mass of neutron

$$c = 3 \times 10^8 \text{ m/s}$$

$$k_B = 1.380 \times 10^{-23} \text{ J/K}$$

$$h = 6.626 \times 10^{-34} \text{ J.s}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N.m}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$m_e = 9.31 \times 10^{-31} \text{ kg}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$m_n = 1.675 \times 10^{-27} \text{ kg}$$

PH (GATE 2014)

**Q. 1 – Q. 25 carry one mark each.**

Q.1 The unit vector perpendicular to the surface  $x^2 + y^2 + z^2 = 3$  at the point (1,1,1) is

- (A)  $\frac{\hat{x} + \hat{y} - \hat{z}}{\sqrt{3}}$       (B)  $\frac{\hat{x} - \hat{y} - \hat{z}}{\sqrt{3}}$       (C)  $\frac{\hat{x} - \hat{y} + \hat{z}}{\sqrt{3}}$       (D)  $\frac{\hat{x} + \hat{y} + \hat{z}}{\sqrt{3}}$

Q.2 Which one of the following quantities is invariant under Lorentz transformation?

- (A) Charge density      (B) Charge      (C) Current      (D) Electric field

Q.3 The number of normal Zeeman splitting components of  $^1P \rightarrow ^1D$  transition is

- (A) 3      (B) 4      (C) 8      (D) 9

Q.4 If the half-life of an elementary particle moving with speed  $0.9c$  in the laboratory frame is  $5 \times 10^{-8} s$ , then the proper half-life is \_\_\_\_\_  $\times 10^{-8} s$ . ( $c = 3 \times 10^8 m/s$ )

Q.5 An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is

- (A)  $0^\circ$       (B)  $45^\circ$       (C)  $90^\circ$       (D)  $120^\circ$

Q.6 Two masses  $m$  and  $3m$  are attached to the two ends of a massless spring with force constant  $K$ . If  $m = 100 g$  and  $K = 0.3 N/m$ , then the natural angular frequency of oscillation is \_\_\_\_\_  $Hz$ .

Q.7 The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by,

$$\vec{E} = \hat{x} 10 \cos(6\pi \times 10^7 t - 0.4\pi z) \text{ V/m.}$$

The phase velocity of the wave is \_\_\_\_\_  $\times 10^8 m/s$ .

Q.8 The matrix

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \text{ is}$$

- (A) orthogonal      (B) symmetric      (C) anti-symmetric      (D) unitary

Q.9 The recoil momentum of an atom is  $p_A$  when it emits an infrared photon of wavelength  $1500 nm$ , and it is  $p_B$  when it emits a photon of visible wavelength  $500 nm$ . The ratio  $\frac{p_A}{p_B}$  is

- (A) 1:1      (B)  $1:\sqrt{3}$       (C) 1:3      (D) 3:2

Q.10 For a gas under isothermal conditions, its pressure  $P$  varies with volume  $V$  as  $P \propto V^{-5/3}$ . The bulk modulus  $B$  is proportional to

- (A)  $V^{-1/2}$       (B)  $V^{-2/3}$       (C)  $V^{-3/5}$       (D)  $V^{-5/3}$

Q.11 Which one of the following high energy processes is allowed by conservation laws?

- (A)  $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$       (B)  $\pi^- + p \rightarrow \pi^0 + n$   
 (C)  $n \rightarrow p + e^- + \nu_e$       (D)  $\mu^+ \rightarrow e^+ + \gamma$



Q.12 The length element  $ds$  of an arc is given by,  $(ds)^2 = 2(dx^1)^2 + (dx^2)^2 + \sqrt{3} dx^1 dx^2$ . The metric tensor  $g_{ij}$  is

(A)  $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

(B)  $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$

(C)  $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$

(D)  $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

Q.13 The ground state and the first excited state wave functions of a one dimensional infinite potential well are  $\psi_1$  and  $\psi_2$ , respectively. When two spin-up electrons are placed in this potential, which one of the following, with  $x_1$  and  $x_2$  denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(A)  $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)]$

(B)  $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)]$

(C)  $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)]$

(D)  $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$

Q.14 If the vector potential

$$\vec{A} = \alpha x \hat{x} + 2y \hat{y} - 3z \hat{z},$$

satisfies the Coulomb gauge, the value of the constant  $\alpha$  is \_\_\_\_\_.

Q.15 At a given temperature,  $T$ , the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillators is \_\_\_\_\_  $k_B T$  ( $k_B$  is the Boltzmann constant).

Q.16 Which one of the following is a fermion?

(A)  $\alpha$  particle

(B)  ${}^7_4\text{Be}$  nucleus

(C) Hydrogen atom

(D) Deuteron

Q.17 Which one of the following three-quark states ( $qqq$ ), denoted by  $X$ , CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript.

(A)  $X^{++}$

(B)  $X^+$

(C)  $X^-$

(D)  $X^{--}$

Q.18 The Hamilton's canonical equations of motion in terms of Poisson Brackets are

(A)  $\dot{q} = \{q, H\}; \dot{p} = \{p, H\}$

(B)  $\dot{q} = \{H, q\}; \dot{p} = \{H, p\}$

(C)  $\dot{q} = \{H, p\}; \dot{p} = \{H, q\}$

(D)  $\dot{q} = \{p, H\}; \dot{p} = \{q, H\}$

Q.19 The Miller indices of a plane passing through the three points having coordinates  $(0,0,1)$ ,  $(1,0,0)$ ,  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{4})$  are

(A) (212)

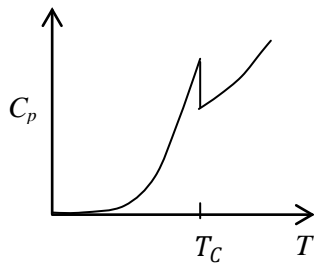
(B) (111)

(C) (121)

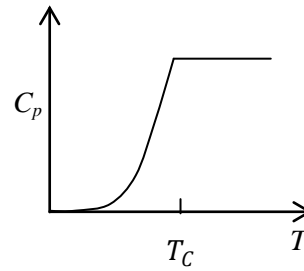
(D) (211)

Q.20 The plot of specific heat versus temperature across the superconducting transition temperature ( $T_C$ ) is most appropriately represented by

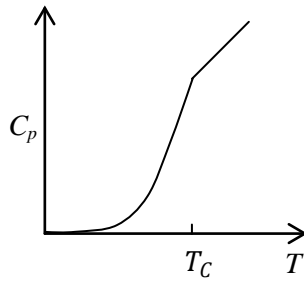
(A)



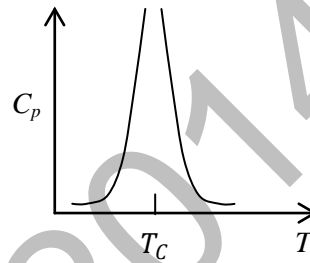
(B)



(C)



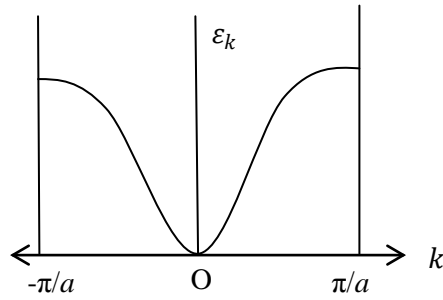
(D)



Q.21 If  $\vec{L}$  is the orbital angular momentum and  $\vec{S}$  is the spin angular momentum, then  $\vec{L} \cdot \vec{S}$  does NOT commute with

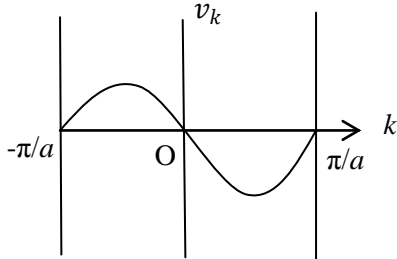
(A)  $S_z$ (B)  $L^2$ (C)  $S^2$ (D)  $(\vec{L} + \vec{S})^2$

- Q.22 The energy,  $\epsilon_k$  for band electrons as a function of the wave vector,  $k$  in the first Brillouin zone ( $-\pi/a \leq k \leq \pi/a$ ) of a one dimensional monatomic lattice is shown as ( $a$  is lattice constant)

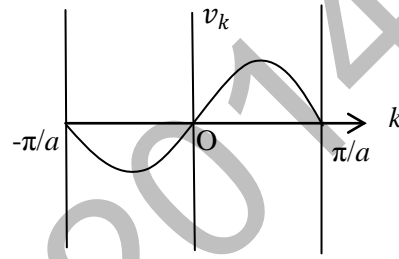


The variation of the group velocity  $v_k$  is most appropriately represented by

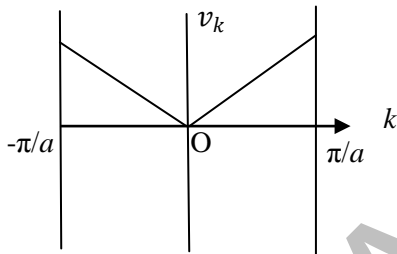
(A)



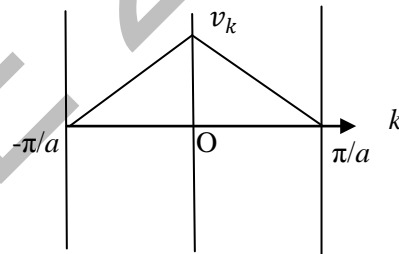
(B)



(C)

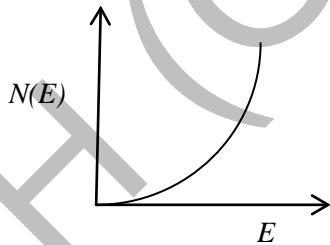


(D)

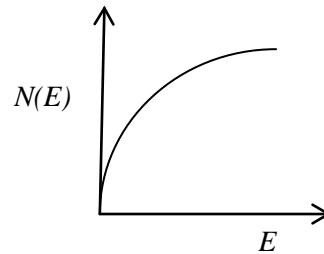


- Q.23 For a free electron gas in two dimensions, the variation of the density of states,  $N(E)$  as a function of energy  $E$ , is best represented by

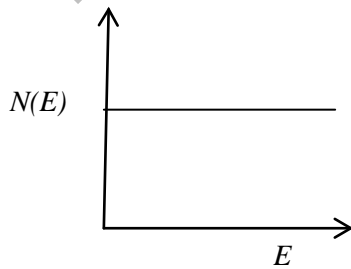
(A)



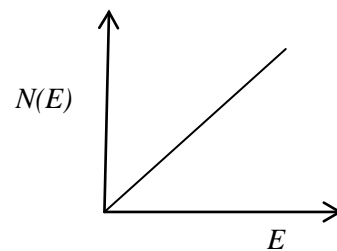
(B)



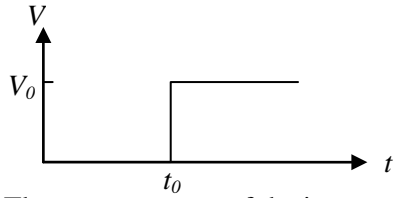
(C)



(D)

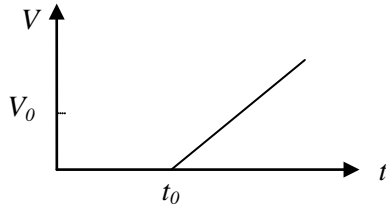


Q.24 The input given to an ideal OP-AMP integrator circuit is

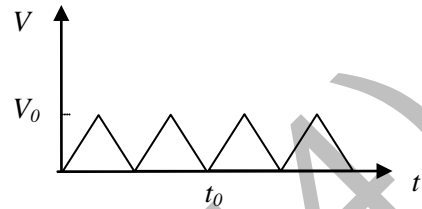


The correct output of the integrator circuit is

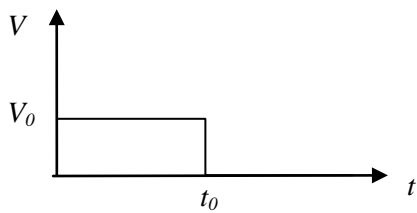
(A)



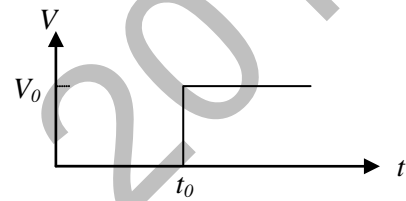
(B)



(C)



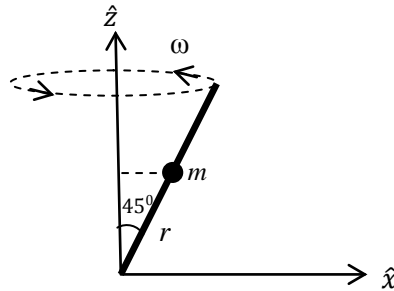
(D)



Q.25 The minimum number of flip-flops required to construct a mod-75 counter is \_\_\_\_\_.

**Q. 26 – Q. 55 carry two marks each.**

- Q.26 A bead of mass  $m$  can slide without friction along a massless rod kept at  $45^\circ$  with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed  $\omega$ . At any instant,  $r$  is the distance of the bead from the origin. The momentum conjugate to  $r$  is



- (A)  $m\dot{r}$                       (B)  $\frac{1}{\sqrt{2}}m\dot{r}$                       (C)  $\frac{1}{2}m\dot{r}$                       (D)  $\sqrt{2}m\dot{r}$

- Q.27 An electron in the ground state of the hydrogen atom has the wave function

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where  $a_0$  is constant. The expectation value of the operator  $\hat{Q} = z^2 - r^2$ , where  $z = r \cos \theta$  is

(Hint:  $\int_0^\infty e^{-\alpha r} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+1}}$ )

- (A)  $-a_0^2/2$                       (B)  $-a_0^2$                       (C)  $-3a_0^2/2$                       (D)  $-2a_0^2$

- Q.28 For Nickel, the number density is  $8 \times 10^{23} \text{ atoms/cm}^3$  and electronic configuration is  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$ . The value of the saturation magnetization of Nickel in its ferromagnetic state is \_\_\_\_\_  $\times 10^9 \text{ A/m}$ .

(Given the value of Bohr magneton  $\mu_B = 9.21 \times 10^{-21} \text{ Am}^2$ )

- Q.29 A particle of mass  $m$  is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3},$$

where  $a$  and  $r_0$  are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

- (A)  $2\pi\sqrt{\frac{mr_0^3}{2a}}$                       (B)  $2\pi\sqrt{\frac{mr_0^3}{a}}$                       (C)  $2\pi\sqrt{\frac{2mr_0^3}{a}}$                       (D)  $4\pi\sqrt{\frac{mr_0^3}{a}}$

- Q.30 The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non degenerate is \_\_\_\_\_  $\text{meV}$ .

( $k_B T = 25 \text{ meV}$  at 300 K)

- Q.31 A particle of mass  $m$  is subjected to a potential,

$$V(x, y) = \frac{1}{2}m\omega^2(x^2 + y^2), -\infty \leq x \leq \infty, -\infty \leq y \leq \infty$$

The state with energy  $4\hbar\omega$  is  $g$ -fold degenerate. The value of  $g$  is \_\_\_\_\_.

Q.32 A hydrogen atom is in the state

$$\Psi = \sqrt{\frac{8}{21}} \psi_{200} - \sqrt{\frac{3}{7}} \psi_{310} + \sqrt{\frac{4}{21}} \psi_{321},$$

where  $n, l, m$  in  $\psi_{nlm}$  denote the principal, orbital and magnetic quantum numbers, respectively. If  $\vec{L}$  is the angular momentum operator, the average value of  $L^2$  is \_\_\_\_\_  $\hbar^2$ .

Q.33 A planet of mass  $m$  moves in a circular orbit of radius  $r_0$  in the gravitational potential  $V(r) = -\frac{k}{r}$ , where  $k$  is a positive constant. The orbital angular momentum of the planet is

- (A)  $2r_0km$                       (B)  $\sqrt{2r_0km}$                       (C)  $r_0km$                       (D)  $\sqrt{r_0km}$

Q.34 The moment of inertia of a rigid diatomic molecule  $A$  is 6 times that of another rigid diatomic molecule  $B$ . If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers  $J_A$  and  $J_B$  are

- (A)  $J_A = 2, J_B = 1$                       (B)  $J_A = 3, J_B = 1$                       (C)  $J_A = 5, J_B = 0$                       (D)  $J_A = 6, J_B = 1$

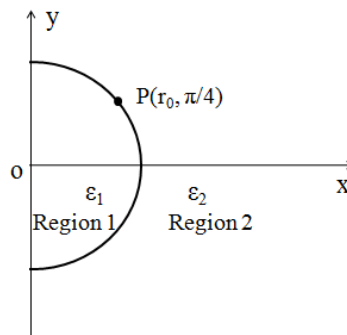
Q.35 The value of the integral

$$\oint_C \frac{z^2}{e^z + 1} dz,$$

where  $C$  is the circle  $|z| = 4$ , is

- (A)  $2\pi i$                       (B)  $2\pi^2 i$                       (C)  $4\pi^3 i$                       (D)  $4\pi^2 i$

Q.36 A ray of light inside Region 1 in the  $xy$ - plane is incident at the semicircular boundary that carries no free charges. The electric field at the point  $P(r_0, \pi/4)$  in plane polar coordinates is  $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\phi$ , where  $\hat{e}_r$  and  $\hat{e}_\phi$  are the unit vectors. The emerging ray in Region 2 has the electric field  $\vec{E}_2$  parallel to  $x$ - axis. If  $\epsilon_1$  and  $\epsilon_2$  are the dielectric constants of Region 1 and Region 2 respectively, then  $\frac{\epsilon_2}{\epsilon_1}$  is \_\_\_\_\_



Q.37 The solution of the differential equation

$$\frac{d^2 y}{dt^2} - y = 0,$$

subject to the boundary conditions  $y(0) = 1$  and  $y(\infty) = 0$ , is

- (A)  $\cos t + \sin t$       (B)  $\cosh t + \sinh t$       (C)  $\cos t - \sin t$       (D)  $\cosh t - \sinh t$

Q.38 Given that the linear transformation of a generalized coordinate  $q$  and the corresponding momentum  $p$ ,

$$\begin{aligned} Q &= q + 4ap \\ P &= p + 2p \end{aligned}$$

is canonical, the value of the constant  $a$  is \_\_\_\_\_.

Q.39 The value of the magnetic field required to maintain non-relativistic protons of energy  $1 \text{ MeV}$  in a circular orbit of radius  $100 \text{ mm}$  is \_\_\_\_\_ Tesla.

(Given:  $m_p = 1.67 \times 10^{-27} \text{ kg}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ )

Q.40 For a system of two bosons, each of which can occupy any of the two energy levels  $0$  and  $\varepsilon$ , the mean energy of the system at a temperature  $T$  with  $\beta = \frac{1}{k_B T}$  is given by

(A)  $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(B)  $\frac{1 + \varepsilon e^{-\beta\varepsilon}}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(C)  $\frac{2\varepsilon e^{-\beta\varepsilon} + \varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(D)  $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

Q.41 In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are  $9I_0$  and  $I_0$ , respectively. The intensities of the individual waves are

(A)  $3I_0$  and  $I_0$

(B)  $4I_0$  and  $I_0$

(C)  $5I_0$  and  $4I_0$

(D)  $9I_0$  and  $I_0$

Q.42  $\psi_1$  and  $\psi_2$  are two orthogonal states of a spin  $\frac{1}{2}$  system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{\frac{2}{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  represent the spin-up and spin-down states, respectively. When the system is in the state  $\psi_2$ , its probability to be in the spin-up state is \_\_\_\_\_.

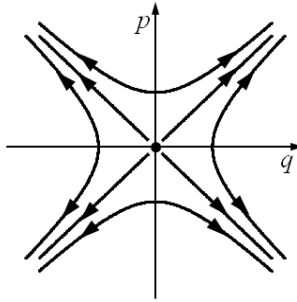
Q.43 Neutrons moving with speed  $10^3 \text{ m/s}$  are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is  $30^\circ$ , the interplanar spacing of the crystal is \_\_\_\_\_ Å.

(Given:  $m_n = 1.675 \times 10^{-27} \text{ kg}$ ,  $h = 6.626 \times 10^{-34} \text{ J.s}$ )

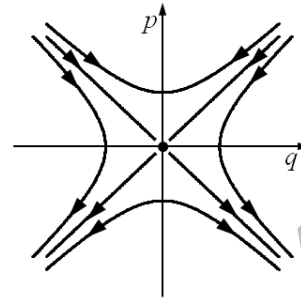
Q.44

The Hamiltonian of a particle of mass  $m$  is given by  $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$ . Which one of the following figures describes the motion of the particle in phase space?

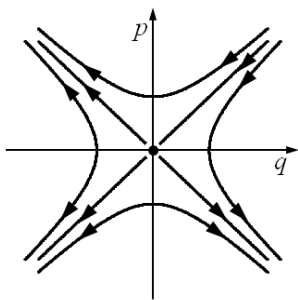
(A)



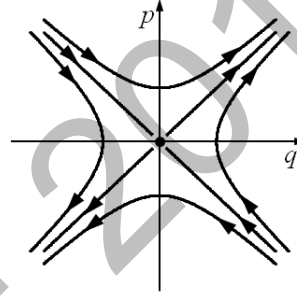
(B)



(C)



(D)



Q.45 The intensity of a laser in free space is  $150 \text{ mW/m}^2$ . The corresponding amplitude of the electric field of the laser is \_\_\_\_\_  $\text{V/m}$ . ( $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N.m}^2$ )

Q.46 The emission wavelength for the transition  ${}^1D_2 \rightarrow {}^1F_3$  is  $3122 \text{ \AA}$ . The ratio of populations of the final to the initial states at a temperature  $5000 \text{ K}$  is ( $h = 6.626 \times 10^{-34} \text{ J.s}$ ,  $c = 3 \times 10^8 \text{ m/s}$ ,  $k_B = 1.380 \times 10^{-23} \text{ J/K}$ )

- (A)  $2.03 \times 10^{-5}$       (B)  $4.02 \times 10^{-5}$       (C)  $7.02 \times 10^{-5}$       (D)  $9.83 \times 10^{-5}$

Q.47 Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

- (A)  $k_B \ln 2$       (B)  $2k_B \ln 2$       (C)  $2k_B \ln 4$       (D)  $3k_B \ln 4$

Q.48 A particle is confined to a one dimensional potential box with the potential

$$V(x) = 0, \quad 0 < x < a$$

$$= \infty, \quad \text{otherwise}$$

If the particle is subjected to a perturbation, within the box,  $W = \beta x$ , where  $\beta$  is a small constant, the first order correction to the ground state energy is

- (A) 0      (B)  $a\beta/4$       (C)  $a\beta/2$       (D)  $a\beta$

Q.49 Consider the process  $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$ . The minimum kinetic energy of the muons ( $\mu$ ) in the centre of mass frame required to produce the pion ( $\pi$ ) pairs at rest is \_\_\_\_\_  $\text{MeV}$ . (Given:  $m_\mu = 105 \text{ MeV}/c^2$ ,  $m_\pi = 140 \text{ MeV}/c^2$ ).



- Q.50 A one dimensional harmonic oscillator is in the superposition of number states,  $|n\rangle$ , given by

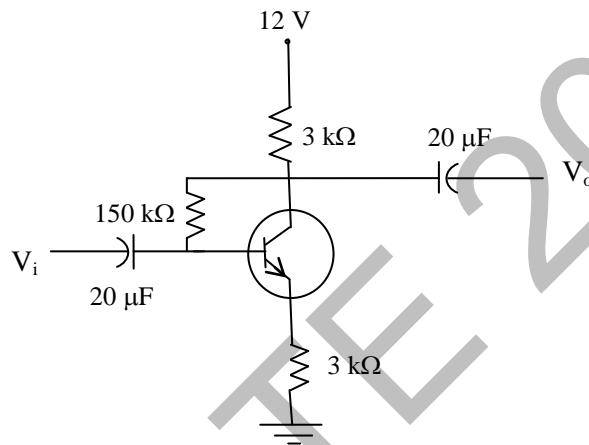
$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle.$$

The average energy of the oscillator in the given state is \_\_\_\_\_  $\hbar\omega$ .

- Q.51 A nucleus  $X$  undergoes a first forbidden  $\beta$ -decay to a nucleus  $Y$ . If the angular momentum ( $I$ ) and parity ( $P$ ), denoted by  $I^P$  as  $\frac{7^-}{2}$  for  $X$ , which of the following is a possible  $I^P$  value for  $Y$ ?

- (A)  $\frac{1^+}{2}$                       (B)  $\frac{1^-}{2}$                       (C)  $\frac{3^+}{2}$                       (D)  $\frac{3^-}{2}$

- Q.52 The current gain of the transistor in the following circuit is  $\beta_{dc} = 100$ . The value of collector current  $I_c$  is \_\_\_\_\_  $mA$ .



- Q.53 In order to measure a maximum of  $1V$  with a resolution of  $1mV$  using a  $n$ -bit  $A/D$  converter, working under the principle of ladder network, the minimum value of  $n$  is \_\_\_\_\_ .
- Q.54 If  $L_+$  and  $L_-$  are the angular momentum ladder operators, then, the expectation value of  $(L_+L_- + L_-L_+)$ , in the state  $|l = 1, m = 1\rangle$  of an atom is \_\_\_\_\_  $\hbar^2$  .
- Q.55 A low pass filter is formed by a resistance  $R$  and a capacitance  $C$ . At the cut-off angular frequency  $\omega_c = \frac{1}{RC}$ , the voltage gain and the phase of the output voltage relative to the input voltage respectively, are

- (A)  $0.71$  and  $45^0$                       (B)  $0.71$  and  $-45^0$                       (C)  $0.5$  and  $-90^0$                       (D)  $0.5$  and  $90^0$

**END OF THE QUESTION PAPER**